be giving us something for nothing, which can't be true. As the contact point moves left and right, the point of application of the force between cam and follower moves with it. There is an overturning moment on the follower associated with this off-center force that tends to jam the follower in its guides, just as did too large a pressure angle in the roller follower case. In this case, we would like to keep the cam as small as possible in order to minimize the moment arm of the force. Eccentricity will affect the average value of the moment, but the peak-to-peak variation of the moment about that average is unaffected by eccentricity. Considerations of a too-large pressure angle do not limit the size of this cam, but other factors do. The minimum radius of curvature of the cam surface must be kept large enough to avoid undercutting (see Figures 7-10 and 7-11 and their discussion on pp. 163-164). This is true regardless of the type of follower used.

### 7.5 PRESSURE ANGLE—RADIAL CAM WITH OSCILLATING ROLLER FOLLOWER

The geometry of a radial cam with an oscillating arm roller follower is shown in Figure 7-8. Derivations for the pressure angle in this case have been developed by Baxter, ${ }^{[1]}$


FIGURE 7-8
Geometry for derivation of pressure angle in a radial cam with oscillating roller follower

Kloomock and Muffley, ${ }^{[2]}$ and Raven. ${ }^{[3]}$ We will follow Kloomock and Muffley's presentation here. Figure 7-8 shows two positions of the follower arm $B C$ being rotated around a "stationary" cam in the typical inversion of the motion for analysis purposes. (Typically, the follower arm pivot $B$ remains stationary and the cam rotates.) The initial position $B C$ becomes $B^{\prime} C^{\prime}$ at a later time after the cam has rotated through the angle $\gamma$. Though these positions are shown widely separated for clarity, the analysis considers them to be an infinitesimal angle $d \gamma$ apart.

The pressure angle $\phi$ is defined as the angle between the normal force $N$ applied at the cam-roller interface, shown as vector $C^{\prime} N$, and the direction of the velocity of the roller center, shown as $C^{\prime} D^{\prime}$. Neglecting friction and taking moments about the arm pivot $B^{\prime}$ gives

$$
\begin{equation*}
\frac{N l}{T}=\frac{1}{\cos \phi} \tag{7.6}
\end{equation*}
$$

where $l$ is the length of the arm and $T$ is the applied load torque on the follower arm. The torque ratio $N / / T$ is similar to the force magnification factor $N / F$ of equation 7.1 b for a radial cam with translating roller follower.

From the geometry of Figure $7-8$, note that as $\gamma$ approaches zero, $\delta^{\prime}$ approaches $\delta$, and $\varepsilon^{\prime}$ approaches $\varepsilon$. An expression for pressure angle $\phi$ can be written as:

$$
\begin{align*}
& \phi=\frac{\pi}{2}-\varepsilon+\lambda  \tag{7.7a}\\
& \lambda=\tan ^{-1} \frac{1}{R} \frac{d R}{d \gamma} \tag{7.7b}
\end{align*}
$$

The triangle $O B^{\prime} C^{\prime}$ in Figure 7-8a (and shown separately in Figure 7-8b) can be solved for $R, \varepsilon$, and $\psi$.

$$
\begin{align*}
R & =\sqrt{l^{2}+c^{2}-2 l c \cos \delta}  \tag{7.7c}\\
\varepsilon & =\sin ^{-1}\left(\frac{c}{R} \sin \delta\right)  \tag{7.7d}\\
\psi & =\cos ^{-1}\left(\frac{c^{2}+R^{2}-l^{2}}{2 R c}\right) \tag{7.7e}
\end{align*}
$$

Also from Figure 7-8 it can be seen that

$$
\begin{equation*}
\gamma=\psi_{0}-\psi+\theta \tag{7.7f}
\end{equation*}
$$

Differentiating equation 7.7 f with respect to $R$ :

$$
\begin{equation*}
\frac{d \gamma}{d R}=\frac{d \theta}{d R}-\frac{d \psi}{d R} \tag{7.7~g}
\end{equation*}
$$

Differentiating equation 7.7 c with respect to $\theta$ and reciprocating:

$$
\begin{equation*}
\frac{d \theta}{d R}=\frac{R}{l c \sin \delta \frac{d \delta}{d \theta}} \tag{7.7h}
\end{equation*}
$$

Differentiating equation 7.7e with respect to $R$ :

$$
\begin{equation*}
\frac{d \psi}{d R}=\frac{c^{2}-R^{2}-l^{2}}{2 R^{2} c \sin \psi} \tag{7.7i}
\end{equation*}
$$

Collecting terms from equations $7.7 \mathrm{a}, 7.7 \mathrm{~d}, 7.7 \mathrm{~g}, 7.7 \mathrm{~h}$, and 7.7 i , and substituting in equation 7.7b gives the following expressions for pressure angle during a rise or fall.

$$
\begin{aligned}
& \phi_{1}=\frac{\pi}{2}-\sin ^{-1}\left(\frac{c}{R} \sin \delta\right)+\tan ^{-1}\left(\frac{1}{\frac{R^{2}}{l c \sin \delta \frac{d \delta}{d \theta}}-\frac{c^{2}-R^{2}-l^{2}}{2 R c \sin \psi}}\right) \\
& \phi_{2}=-\frac{\pi}{2}+\sin ^{-1}\left(\frac{c}{R} \sin \delta\right)+\tan ^{-1}\left(\frac{1}{\frac{R^{2}}{l c \sin \delta \frac{d \delta}{d \theta}}+\frac{c^{2}-R^{2}-l^{2}}{2 R c \sin \psi}}\right)
\end{aligned}
$$

where $\delta$ is the lift angle of the arm for each cam position with respect to the line of centers $C B$ between cam and follower arm pivot (in radians), $\varepsilon$ is from equation 7.7 d , and $d \delta / d \theta$ is the angular velocity $v$ of the follower arm for each cam position (in rad/rad, i.e., dimensionless). The angle $\psi$ varies with each cam position due to the arc motion of the follower tip. This geometry must be computed for each position of the cam for which a value of the pressure angle is desired.

The first expression for $\phi$ (equation 7.8a) is used for the case of the cam rotating away from the follower arm pivot as depicted in Figure 7-8. If the cam shown in that figure were rotating clockwise, it would be rotating toward the follower arm pivot and then equation 7.8 b must be used. Note that these two equations differ only by three signs.

When the follower is on a dwell, the equations for pressure angle become:

$$
\begin{align*}
& \phi_{1}=\frac{\pi}{2}-\sin ^{-1}\left(\frac{c}{R} \sin \delta\right)  \tag{7.8c}\\
& \phi_{2}=-\frac{\pi}{2}+\sin ^{-1}\left(\frac{c}{R} \sin \delta\right) \tag{7.8d}
\end{align*}
$$

with equation 7.8 c used for the case of the cam rotating away from, and 7.8 d for the case of the cam rotating toward, the follower arm pivot. Equation 7.7d defines $\varepsilon$.

