$$\mathbf{W} = we^{j\theta}; \qquad \mathbf{Z} = ze^{j\phi} \tag{5.32d}$$

and:

$$\begin{aligned} \mathbf{W} + \mathbf{Z} &= \mathbf{R}_1 \\ \mathbf{W}e^{j\beta_2} + \mathbf{Z}e^{j\alpha_2} &= \mathbf{R}_2 \\ \mathbf{W}e^{j\beta_3} + \mathbf{Z}e^{j\alpha_3} &= \mathbf{R}_3 \end{aligned} \tag{5.32e}$$

Previously, we chose β_2 and β_3 and solved for the vectors **W** and **Z**. Now we wish to, in effect, specify the x, y components of the fixed pivot O_2 ($-R_{1x}$, $-R_{1y}$) as our two free choices. This leaves β_2 and β_3 to be solved for. These angles are contained in transcendental expressions in the equations. Note that, if we assumed values for β_2 and β_3 as before, there could only be a solution for **W** and **Z** if the determinant of the augmented matrix of coefficients of equations 5.32e were equal to zero.

$$\begin{bmatrix} 1 & 1 & \mathbf{R}_1 \\ e^{j\beta_2} & e^{j\alpha_2} & \mathbf{R}_2 \\ e^{j\beta_3} & e^{j\alpha_3} & \mathbf{R}_3 \end{bmatrix} = 0 \tag{5.33a}$$

Expand this determinant about the first column which contains the present unknowns β_2 and β_3:

$$\left(\mathbf{R}_3 e^{j\alpha_2} - \mathbf{R}_2 e^{j\alpha_3}\right) + e^{j\beta_2}\left(\mathbf{R}_1 e^{j\alpha_3} - \mathbf{R}_3\right) + e^{j\beta_3}\left(\mathbf{R}_2 - \mathbf{R}_1 e^{j\alpha_2}\right) = 0 \tag{5.33b}$$

To simplify, let:

$$\begin{aligned} A &= \mathbf{R}_3 e^{j\alpha_2} - \mathbf{R}_2 e^{j\alpha_3} \\ B &= \mathbf{R}_1 e^{j\alpha_3} - \mathbf{R}_3 \\ C &= \mathbf{R}_2 - \mathbf{R}_1 e^{j\alpha_2} \end{aligned} \tag{5.33c}$$

then:

$$A + Be^{j\beta_2} + Ce^{j\beta_3} = 0 \tag{5.33d}$$

Equation 5.33d expresses the summation of vectors around a closed loop. Angles β_2 and β_3 are contained within transcendental expressions making their solution cumbersome. The procedure is similar to that used for the analysis of the fourbar linkage in Section 4.5 (p. 171). Substitute the complex number equivalents for all vectors in equation 5.33d. Expand using the Euler identity (equation 4.4a, p. 173). Separate real and imaginary terms to get two simultaneous equations in the two unknowns β_2 and β_3. Square these expressions and add them to eliminate one unknown. Simplify the resulting mess and substitute the tangent half angle identities to get rid of the mixture of sines and cosines. It will ultimately reduce to a quadratic equation in the tangent of half the angle sought, here β_3. β_2 can then be found by back substituting β_3 in the original equations. The results are:*

$$\begin{aligned} \beta_3 &= 2\arctan\left(\frac{K_2 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \\ \beta_2 &= \arctan\left[\frac{-\left(A_3 \sin\beta_3 + A_2 \cos\beta_3 + A_4\right)}{-\left(A_5 \sin\beta_3 + A_3 \cos\beta_3 + A_6\right)}\right] \end{aligned} \tag{5.34a}$$

* Note that a two-argument arctangent function must be used to obtain the proper quadrants for angles β_2 and β_3. Also, the minus signs in numerator and denominator of the equation for β_2 look like they could be canceled, but should not be. They are needed to determine the correct quadrant of β_2 in the two-argument arctangent function.

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