5.1 FAILURE OF DUCTILE MATERIALS UNDER STATIC LOADING

While ductile materials will fracture if statically stressed beyond their ultimate tensile strength, their failure in machine parts is generally considered to occur when they yield under static loading. The yield strength of a ductile material is appreciably less than its ultimate strength.

Historically, several theories have been formulated to explain this failure: *the maximum normal-stress theory, the maximum normal-strain theory, the total strain-energy theory, the distortion-energy (von Mises-Hencky) theory, and the maximum shear-stress theory*. Of these only the last two agree closely with experimental data for this case, and of those, the von Mises-Hencky theory is the most accurate. We will discuss only the last two in detail, starting with the most accurate (and preferred) approach.

The von Mises-Hencky or Distortion-Energy Theory

The microscopic yielding mechanism is now understood to be due to relative sliding of the material's atoms within their lattice structure. This sliding is caused by shear stress and is accompanied by distortion of the shape of the part. The energy stored in the part from this distortion is an indicator of the magnitude of the shear stress present.

TOTAL STRAIN ENERGY It was once thought that the total strain energy stored in the material was the cause of yield failure, but experimental evidence did not bear this out. The strain energy U in a unit volume (strain energy density) associated with any stress is the area under the stress-strain curve up to the point of the applied stress, as shown in Figure 5-2 for a unidirectional stress state. Assuming that the stress-strain curve is essentially linear up to the yield point, then we can express the total strain energy in a unit volume at any point in that range as

$$U = \frac{1}{2}\sigma\varepsilon \tag{5.1a}$$

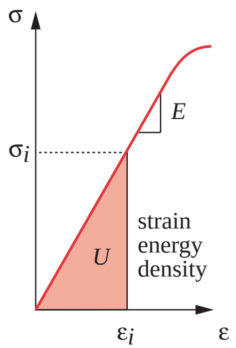

FIGURE 5-2

Internal Strain Energy Density in a Deflected Part

Extending this to a three-dimensional stress state gives

$$U = \frac{1}{2}\left(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right) \tag{5.1b}$$

using the principal stresses and principal strains that act on planes of zero shear stress.

This expression can be put in terms of principal stresses alone by substituting the relationships

$$\begin{aligned}
\varepsilon_1 &= \frac{1}{E}\left(\sigma_1 - \nu\sigma_2 - \nu\sigma_3\right)\\
\varepsilon_2 &= \frac{1}{E}\left(\sigma_2 - \nu\sigma_1 - \nu\sigma_3\right)\\
\varepsilon_3 &= \frac{1}{E}\left(\sigma_3 - \nu\sigma_1 - \nu\sigma_2\right)
\end{aligned} \tag{5.1c}$$