

FIGURE 5-17

Three Modes of Crack Displacement

speciation. (Many commercial aircraft currently flying contain structural cracks.) These decisions can now be made sensibly through the use of fracture-mechanics theory.

Fracture-Mechanics Theory

Fracture mechanics presumes the presence of a crack. The stress state in the region of the crack may be one of plane strain or plane stress (see Section 4.4, p. 150). If the zone of yielding around the crack is small compared to the dimensions of the part, then **linear-elastic fracture-mechanics** (LEFM) theory is applicable. LEFM assumes that the bulk of the material is behaving according to Hooke's law. However, if a significant portion of the bulk material is in the plastic region of its stress-strain behavior, then a more complicated approach is required than that described here. For the following discussion, we will assume that LEFM applies.

MODES OF CRACK DISPLACEMENT Depending on the orientation of the loading versus the crack, the applied loads may tend to pull the crack open in tension (Mode I), shear the crack in-plane (Mode II), or shear (tear) it out-of-plane (Mode III) as shown in Figure 5-17. Most of the fracture-mechanics research and testing has been devoted to the tensile loading case (Mode I), and we will limit our discussion to it.

STRESS INTENSITY FACTOR K Figure 5-18a shows a plate (not to scale) of width $2b$ under tension with a through crack of width $2a$ in the center. The crack is assumed to be sharp at its ends, and b is much larger than a . The crack's cross section is in the xy plane. An r - θ polar coordinate system is also set up in the xy plane with its origin at the crack tip as shown in Figure 5-18b. From the *theory of linear elasticity*, for $b \gg a$ the stresses around the crack tip, expressed as a function of the polar coordinates, are

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \dots \quad (5.13a)$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] + \dots \quad (5.13b)$$

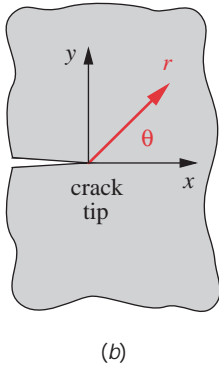


FIGURE 5-18

A Through-Crack in a Plate in Tension