Solution

1 The contact-patch geometry is found in the same way as was done in Example 7-2. Find the material constants from equation 7.9*a* (p. 466).

$$m_1 = m_2 = \frac{1-\nu_1^2}{E_1} = \frac{1-0.28^2}{3E7} = 3.072E-8 \tag{a}$$

The geometry constant is found from equation 7.15*a* (p. 472)

$$B = \frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{1.25}+\frac{1}{2.5}\right) = 0.600 \tag{b}$$

and the patch half-width from equation 7.15*b* (p. 472).

$$a = \sqrt{\frac{2}{\pi}\frac{m_1+m_2}{B}\frac{F}{L}} = \sqrt{\left(\frac{2}{\pi}\right)\frac{2(3.072E-8)}{0.600}\left(\frac{5000}{24}\right)} = 0.003\,685 \text{ in} \tag{c}$$

where a is the half-width of the contact patch. The rectangular contact-patch area is then

$$area = 2aL = 2(0.003\,685)(24) = 0.176\,9 \text{ in}^2 \tag{d}$$

2 The average and maximum contact pressure can now be found from equations 7.14*b* and *c* (p. 472).

$$p_{avg} = \frac{F}{area} = \frac{5000}{0.1769} = 28\,266 \text{ psi} \tag{e}$$

$$p_{max} = \frac{2F}{\pi a L} = \frac{2(5000)}{\pi(0.003\,7)24} = 35\,989 \text{ psi} \tag{f}$$

The tangential pressure is found from equation 7.22*f* (p. 482):

$$f_{max} = \mu\, p_{max} = 0.33(35\,989) = 11\,876 \text{ psi} \tag{g}$$

3 With $\mu = 0.33$, the principal stresses in the contact zone will be maximal on the surface ($z = 0$) at $x = 0.3a$ from the centerline as shown in Figures 7-20 (p. 484) and 7-22 (p. 485). The applied stress components are found from equation 7.23*a* (p. 482) for the normal force and equation 7.23*b* (p. 482) for the tangential force.

$$\begin{aligned} \sigma_{x_n} &= -p_{max}\sqrt{1-\frac{x^2}{a^2}} = -35\,989\sqrt{1-0.3^2} = -34\,331 \text{ psi} \\ \sigma_{x_t} &= -2 f_{max}\frac{x}{a} = -2(11\,876)(0.3) = -7\,126 \text{ psi} \end{aligned} \tag{h}$$

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