frequency will usually provide good results unless the initial eccentricity is excessive (which should not be allowed anyway).

Note the difference between shaft *lateral vibration* and *shaft whirl*. **Lateral vibration** is a *forced vibration*, requiring some outside source of energy such as vibrations from other parts of the machine to precipitate it, and the shaft then vibrates in one or more lateral planes whether or not it is rotating. **Shaft whirl** is a *self-excited vibration* caused by the shaft rotation acting on an eccentric mass. It will *always* occur when both rotation and eccentricity are present. The shaft assumes a deflected shape, which then rotates or whirls about the axis much like a jump-rope being swung by children.

Torsional Vibration

Just as a shaft can vibrate laterally, it can also vibrate torsionally and will have one or more torsional natural frequencies. The same equations that describe lateral vibrations can be used for torsional ones. The systems are analogous. Force becomes torque, mass becomes mass moment of inertia, and linear spring constant becomes torsional spring constant. Equation 9.24 (p. 590) for the circular natural frequency becomes, for a single-degree-of-freedom rotating system:

$$\omega_n = \sqrt{\frac{k_t}{I_m}} \quad \text{rad/sec} \tag{9.27a}$$

The torsional spring constant k_t for a solid circular shaft is

$$k_t = \frac{GJ}{l} \quad \text{lb-in/rad or N-m/rad} \tag{9.27b}$$

where G is the material's modulus of rigidity, and l is the shaft length. The polar second moment of area J of a solid circular shaft is

$$J = \frac{\pi d^4}{32} \quad \text{in}^4 \text{ or } \text{m}^4 \tag{9.27c}$$

If the shaft is stepped, then an equivalent polar second moment of area J_{eff} is found from

$$J_{eff} = \frac{l}{\sum_{i=1}^{n} \frac{l_i}{J_i}} \tag{9.27d}$$

where l is total shaft length, and J_i, and l_i are the polar moments and lengths of the subsections of shaft of differing diameters, respectively.

The mass moment of inertia of a solid circular disk about its axis of rotation is:

$$I_m = \frac{mr^2}{2} \quad \text{in-lb-sec}^2 \text{ or kg-m}^2 \tag{9.27e}$$