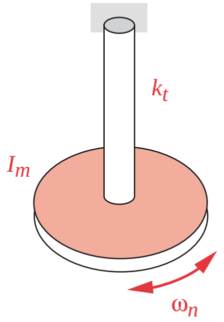

FIGURE 9-30

A Disk on Axle in Torsional Vibration

where r is the disk radius and m is its mass.

These equations are sufficient to find the critical frequency of a single disk mounted on a fixed axle, as shown in Figure 9-30.

Two Disks on a Common Shaft

A more interesting problem is that of two (or more) disks displaced on a common shaft as shown in Figure 9-31. The two disks shown will oscillate torsionally at the same natural frequency, 180° out of phase. There will be a point, called a node, somewhere on the shaft, at which there will be no angular deflection. On either side of the node, points on the shaft rotate in opposite angular directions when vibrating. The system can be modeled as two separate, single-mass systems coupled at this stationary node. One has mass moment and spring constant I_1, k_1 and the other I_2, k_2. Their common natural frequency is then

$$\omega_n = \sqrt{\frac{k_1}{I_1}} = \sqrt{\frac{k_2}{I_2}} \tag{9.28a}$$

The spring constants of the shaft segments are each found from $k_t = JG / l$ assuming that the J is constant across the node.

$$\sqrt{\frac{JG}{l_1 I_1}} = \sqrt{\frac{JG}{l_2 I_2}}$$

and
$$l_1 I_1 = l_2 I_2 = I_2 \left(l - l_1 \right)$$

so
$$l_1 = \frac{I_2 l}{I_1 + I_2} \tag{9.28b}$$

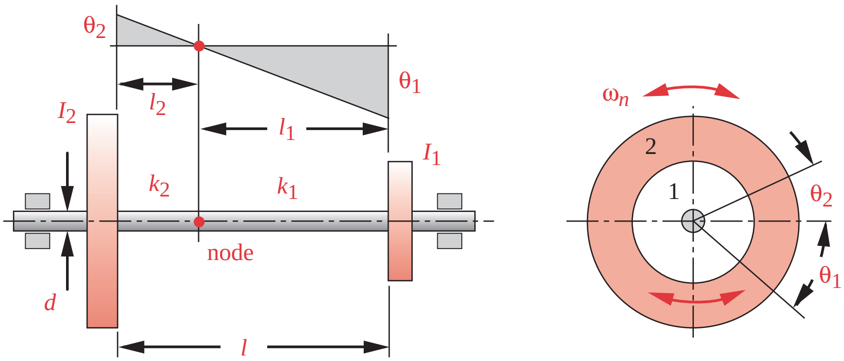

FIGURE 9-31

Torsional Vibration of Two Disks on a Common Shaft