1 Lateral vibration

2 Shaft whirl

3 Torsional vibration

The first two involve bending deflections and the last torsional deflection of the shaft.

Lateral Vibration of Shafts and Beams—Rayleigh's Method

A complete analysis of the natural frequencies of a shaft or beam is a complicated problem, especially if the geometry is complex, and is best solved with the aid of *Finite Element Analysis* software. A so-called **modal analysis** can be done on a finite element model of even complex geometries and will yield a large number of natural frequencies (in three dimensions) from the fundamental up. This is the preferred and frequently used approach when analyzing a completed or mature design in detail. However, in the early stages of design, when the part geometries are still not fully defined, a quick and easily applied method for finding at least an approximate fundamental frequency for a proposed design is very useful. **Rayleigh's method** serves that purpose. It is an energy method that gives results within a few percent of the true ω_n. It can be applied to a continuous system or to a lumped-parameter model of the system. The latter approach is usually preferred for simplicity.

RAYLEIGH'S METHOD equates the potential and kinetic energies in the system. The potential energy is in the form of strain energy in the deflected shaft and is maximum at the largest deflection. The kinetic energy is a maximum when the vibrating shaft passes through the undeflected position with maximum velocity. This method assumes that the lateral vibrating motion of the shaft is sinusoidal and that some external excitation is present to force the lateral vibration (Figure 9-26*a*).

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To illustrate the application of this method, consider a simply-supported shaft with three disks (gears, sheaves, etc.) on it as shown in Figure 9-27. We will model this as three discrete lumps of known mass on a massless shaft. The shaft's geometry will define the bending spring constant, thus lumping all the "spring" into the shaft. The total potential energy stored at maximum deflection is the sum of the potential energies of each lumped mass:

$$E_p = \frac{g}{2}\left(m_1\delta_1 + m_2\delta_2 + m_3\delta_3\right) \tag{9.25a}$$

where the deflections are all taken as positive regardless of the local shape of the deflection curve because the strain energy is not affected by the external coordinate system. The energy of the deflected shaft is ignored as small compared to the disk energy.

The total kinetic energy is the sum of the individual kinetic energies:

$$E_k = \frac{\omega_n^2}{2}\left(m_1\delta_1^2 + m_2\delta_2^2 + m_3\delta_3^2\right) \tag{9.25b}$$

where the velocities are taken as positive.