$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_{fm} \frac{M_m c}{I} \tag{10.2a}$$

where K_f and K_{fm} are the bending-fatigue stress-concentration factors for the alternating and mean components, respectively (see equations 6.11 and 6.17 on pp. 343 and 364). Since the typical shaft is a solid-round cross section,* we can substitute for c and I:

$$c = r = \frac{d}{2} \qquad I = \frac{\pi d^4}{64} \tag{10.2b}$$

giving

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_{fm} \frac{32 M_m}{\pi d^3} \tag{10.2c}$$

where d is the local shaft diameter at the section of interest.

The alternating and mean torsional shear stresses are found from

$$\tau_a = K_{fs} \frac{T_a r}{J} \qquad \tau_m = K_{fsm} \frac{T_m r}{J} \tag{10.3a}$$

where K_{fs} and K_{fsm} are the torsional fatigue stress-concentration factors for the alternating and mean components, respectively (see equation 6.11 on p. 343 for K_{fs} and use the applied shear stresses and shear yield strength in equation 6.17 on p. 364 to get K_{fsm}). For a solid-round cross section,* we can substitute for r and J:

$$r = \frac{d}{2} \qquad J = \frac{\pi d^4}{32} \tag{10.3b}$$

giving

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fsm} \frac{16 T_m}{\pi d^3} \tag{10.3c}$$

A tensile axial load F_z, if any is present, will typically have only a mean component (such as the weight of the components) and can be found from

$$\sigma_{m_{axial}} = K_{fm} \frac{F_z}{A} = K_{fm} \frac{4 F_z}{\pi d^2} \tag{10.4}$$

10.7 SHAFT FAILURE IN COMBINED LOADING

Extensive studies of fatigue failure of both ductile steels and brittle cast irons in combined bending and torsion were done originally in England in the 1930s by Davies[3] and Gough and Pollard.[5] These early results are shown in Figure 10-3, which is taken from the ANSI/ASME Standard B106.1M-1985 on the *Design of Transmission Shafting*. Data from later research is also included on these plots.[2, 4] The combination of torsion and bending on ductile materials in fatigue was found to generally follow the elliptical relationship as defined by the equations in the figure. Cast brittle materials (not shown) were found to fail based on the maximum principal stress. These findings are similar to those for combined torsional and bending stresses in fully reversed loading shown in Figure 6-15 (p. 321).

* For a hollow shaft, substitute the appropriate expressions for *I* and *J*.

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