Table 3–6 - part 1
Case Study 5A
Given and Assumed Data

Variable	Value	Unit
θ_2	30.00	deg
ω_2	120.00	rpm
$mass_2$	0.525	kg
$mass_3$	1.050	kg
$mass_4$	1.050	kg
I_{cg2}	0.057	kg-m^2
I_{cg3}	0.011	kg-m^2
I_{cg4}	0.455	kg-m^2
R_{12x}	–46.9	mm
R_{12y}	–71.3	mm
R_{32x}	85.1	mm
R_{32y}	4.9	mm
R_{23x}	–150.7	mm
R_{23y}	–177.6	mm
R_{43x}	185.5	mm
R_{43y}	50.8	mm
R_{14x}	–21.5	mm
R_{14y}	–100.6	mm
R_{34x}	–10.6	mm
R_{34y}	204.0	mm

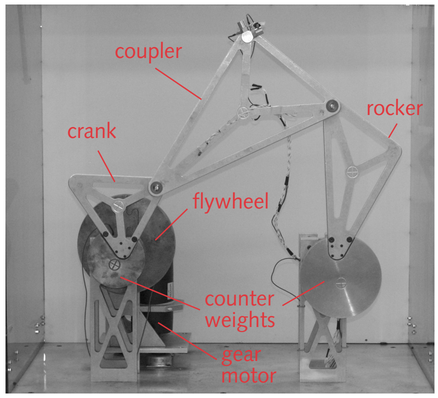

FIGURE 3-11

Fourbar Linkage Dynamic Model

Solution: **See Figures 3-11 through 3-13 and Table 3-6.**

1 Figures 3-11 and 3-12 show the fourbar linkage demonstrator model. It consists of three moving elements (links 2, 3, and 4) plus the frame or ground link (1). The motor drives link 2 through a gearbox. The two fixed pivots are instrumented with piezoelectric force transducers to measure the dynamic forces acting in x and y directions on the ground plane. A pair of accelerometers is mounted to a point on the floating coupler (link 3) to measure its accelerations.

2 Figure 3-12 shows a schematic of the linkage. The links are designed with lightening holes to reduce their masses and mass moments of inertia. The input to link 2 can be an angular acceleration, a constant angular velocity, or an applied torque. Link 2 rotates fully about its fixed pivot at O_2. Even though link 2 may have a zero angular acceleration α_2, if run at constant angular velocity ω_2, there will still be time-varying angular accelerations on links 3 and 4 since they oscillate back and forth. In any case, the CGs of the links will experience time-varying linear accelerations as the linkage moves. These angular and linear accelerations will generate inertia forces and torques

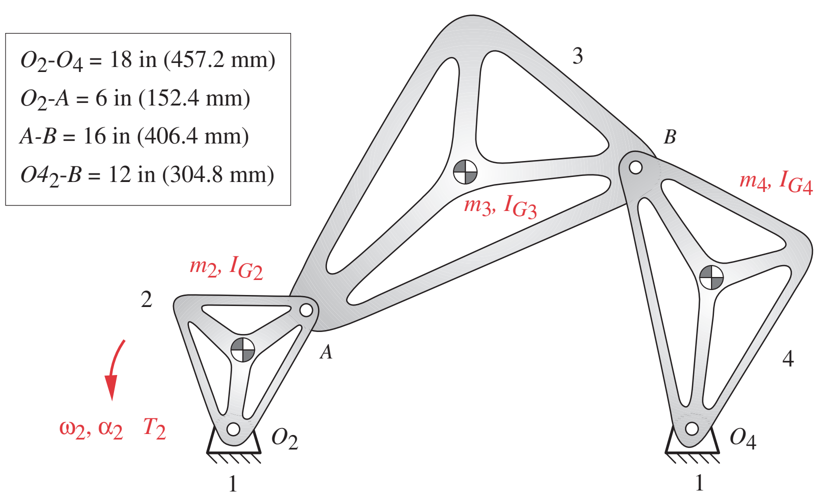

FIGURE 3-12

Fourbar Linkage Schematic and Basic Dimensions (See Table 3-6 for more information)

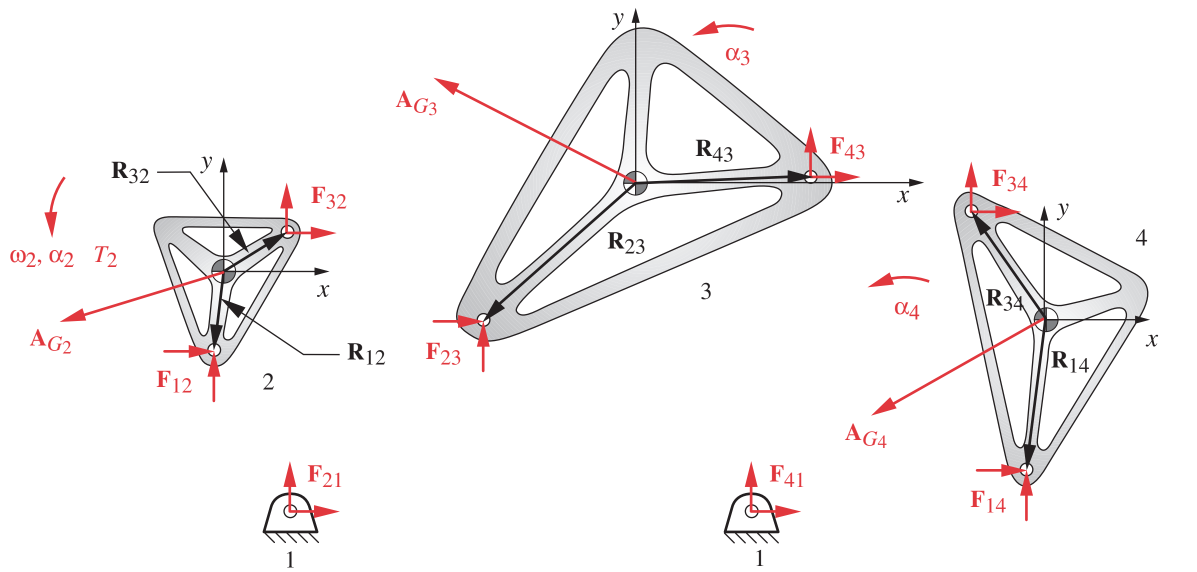

FIGURE 3-13

Free-Body Diagrams of Elements in a Fourbar Linkage

as defined by Newton's second law. Thus, even with no external forces or torques applied to the links, the inertial forces will create reaction forces at the pins. It is these forces that we wish to calculate.

3 Figure 3-13 shows the free-body diagrams of the individual links. The local, nonrotating, coordinate system for each link is set up at its CG. The kinematic equations of motion must be solved to determine the linear accelerations of the CG of each link and the link's angular acceleration for every position of interest during the cycle. (See reference 1 for an explanation of this procedure.) These accelerations, $\mathbf{A}_{Gn}$ and α_n, are shown acting on each of the n links. The forces at each pin connection are shown as xy pairs, numbered as before, and are initially assumed to be positive.

4 Equations 3.1 can be written for each moving link in the system. The masses and the mass moments of inertia of each link about its CG must be calculated for use in these equations. In this case study, a solid modeling CAD system was used to design the links' geometries and to calculate their mass properties.

5 For link 2:

$$\begin{aligned}
\sum F_x &= F_{12x} + F_{32x} = m_2 a_{G2_x} \\
\sum F_y &= F_{12y} + F_{32y} = m_2 a_{G2_y} \\
\sum M_z &= T_2 + \left(R_{12x}F_{12y} - R_{12y}F_{12x}\right) + \left(R_{32x}F_{32y} - R_{32y}F_{32x}\right) = I_{G2}\alpha_2
\end{aligned} \tag{a}$$

6 For link 3:

$$\begin{aligned}
\sum F_x &= F_{23x} + F_{43x} = m_3 a_{G3_x} \\
\sum F_y &= F_{23y} + F_{43y} = m_3 a_{G3_y} \\
\sum M_z &= \left(R_{23x}F_{23y} - R_{23y}F_{23x}\right) + \left(R_{43x}F_{43y} - R_{43y}F_{43x}\right) = I_{G3}\alpha_3
\end{aligned} \tag{b}$$